PERIODIC PROCESSES IN LOCALIZED GAS INJECTION INTO A GRANULAR BED

E. V. Toropov and N. N. Sazhnev UDC 532.529.5

Physical principles are used in formulating conditions for occurrence of acoustic and relaxation oscillations in a bed of granular material in the presence of localized gas injection.

A common industrial technique is the use of a bed of granular material injected locally with a gas (Fig. 1). The gas flows from a vessel 1 at a constant pressure via the nozzle 2 into the bed, where a circulation zone 3 arises, which is relatively free from the granular material. The bed material 4 is displaced downwards by its weight, which is opposed by the mass-transfer processes in zone 3, where combustion, melting, evaporation, etc., may also occur.

The dynamic processes in the gas system and bed are described on the assumption that the hydrodynamic system of Fig. 1 consists of components with lumped parameters — mass, elastic, and dissipative.

The motion over section 1—2 may be described by means of Euler's momentum theorem:

$$\frac{\rho_1 l_1}{s_1}\frac{dQ_1}{d\tau} = (P_1 - P_3) - \Delta P_{1-2}, \tag{1}$$

where the pressure loss over section 1-2 is

$$\Delta P_{1-2} = \left(k_1 + \frac{k_2}{R_3}\right) Q_1^2; \tag{2}$$

and, therefore,

$$Ma_1 \frac{dQ_1}{d\tau} = (P_1 - P_3) - \left(k_1 + \frac{k_2}{R_3}\right) Q_1^2. \tag{3}$$

The acoustic mass of the injected flow is $Ma_1 = \rho_1 l_1/s_1$ in (3), and this characterizes the inertial features of this component.

The gas is compressed in zone 3 (volume V_3) in a reasonably adiabatic fashion, so $\rho^{\varkappa}/P = \mathrm{const}$; differentiation then gives

$$\frac{\varkappa d\rho}{\rho} = \frac{dP}{P}. \tag{4}$$

The density change related to the change in gas speed in the bed can be expressed in terms of the difference between the volume flow rates Q_2 and Q_3 at the inlet and outlet of zone 3:

$$\frac{d\rho}{d\tau} = \frac{\rho}{V_3}(Q_2 - k_3 Q_3). \tag{5}$$

Section 1—2 has only inertial parameters, so $Q_1 = Q_2 (Q = \mathrm{idem})$, and substitution of (5) into (4) along with the equation of state $P = \mu\rho R_g T$ gives

$$\frac{V_3}{\rho c^2}\frac{dP_3}{d\tau} = Q_1 - k_3 Q_3, \tag{6}$$

where $c = \sqrt{\varkappa\mu R_g T}$ is the speed of sound under the conditions of zone 3.

Any change in the free volume V_3 is dependent on the ratio between the rate of input of material to zone 3 and the rate of consumption by reaction:

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$$\frac{dV_3}{d\tau} = k_4 Q_1 - k_5 R_3^2 f(P_3). \tag{7}$$

Here it is assumed that the area through which the material enters the circulation zone is proportional to the square of the relevant linear dimension R_3, while the consumption of the granular material is proportional to the gas flow rate; the function $f(P_3)$ reflects the effects of the pressure P_3 in the zone on the input of material.

The gases pass through the layer surrounding the circulation zone and thereby influence the pressure in that zone, and the acoustic mass Ma_3 of the gas passing through the material then acts as an inertial component:

$$P_3 = \varphi(Q_3) + Ma_3 \frac{dQ_3}{d\tau}. \tag{8}$$

The relationship between V_3 and the size parameter R_3 is

$$V_3 = k_6 R_3^3, \tag{9}$$

and then the dynamic equations can be combined as the system

$$\begin{aligned} Ma_1 \frac{dQ_1}{d\tau} &= (P_1 - P_3) - \left(k_1 + \frac{k_2}{R_3}\right) Q_1^2; \\ Ca_3 \frac{dP_3}{d\tau} &= Q_1 - k_3 Q_3; \\ 3k_6 R_3^2 \frac{dR_3}{d\tau} &= k_4 Q_1 - k_5 R_3^2 f(P_3); \\ P_3 &= \varphi(Q_3) + Ma_3 \frac{dQ_3}{d\tau}. \end{aligned} \tag{10}$$

The function $\varphi(Q_3)$ reflects the pressure drop in the bed above the circulation zone 3, while the structure of the bed is dependent on the initial porosity, the particle packing conditions (themselves affected by the processing), the heat and mass transfer in zone 3 and in the bed, the amounts of gas passing through the bed, and various other factors. There are major difficulties in the theory of jet flows in granular beds even if heat and mass transfer are neglected [1]. A good means of examining processes in such beds is to employ a model that reflects the microscopic or macroscopic parameters of the material that are of major importance for the particular purpose [2-4].

In the present case, the bed is displaced downwards by gravitational forces into the circulation zone, while there are frictional forces on the wall of the apparatus and other immobile parts far from the circulation zone. The friction between the gas flow and the bed is in dynamic equilibrium with the forces between the particles of granular material above the circulation zone.

The following types of structure in the bed occur as Q_3 varies: the close-packed bed opens up for $Q_3 > Q^A{}_3$ and the granular structure becomes apparent; fluidization sets in for $Q_3 > Q^B{}_3$; and for $Q_3 > Q^D{}_3$ the particles are entrained by the gas flow and channels are produced, in which the situation is similar to that of a gas suspension (Fig. 2). The circulation of the material in the case $Q_3 > Q^A{}_3$ is accelerated because of the relatively free movement of the individual particles, while some of the particle cover may be disrupted under fluidization conditions ($Q_3 > Q^B{}_3$), and bubbles of gas may break through the bed. When this occurs, the flow rate then falls to $Q_3 \approx Q^A{}_3$ and the bed reforms, so the elastic behavior of V_3 allows

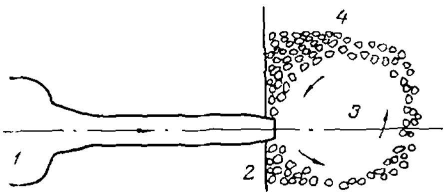

Fig. 1. Process scheme.

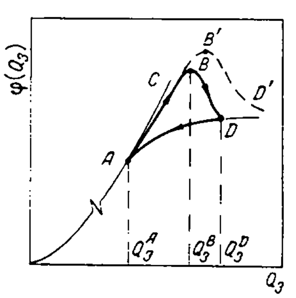

Fig. 2

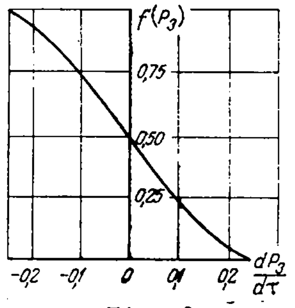

Fig. 3

Fig. 2. The function $\varphi(Q_3)$.

Fig. 3. The function $f(P_3)$ for $\gamma = 16.0$, $\alpha = 3.0$, $\lambda = 0.5$.

the latter to take up the excess potential energy as increased pressure, which is again dissipated in the escape of batches of gas. The area enclosed by the curve ABDA represents the energy required to open up the bed, while part of this energy is consumed when the bed relaxes and accelerates the gas.

Equation (7) contains $f(P_3)$, which reflects the mode of entry of the material into zone 3 as the pressure varies; this can be represented as an incomplete cubic polynomial in $dP_3/d\tau$:

$$f(P_3) = \gamma\left(\frac{dP_3}{d\tau}\right)^3 - \alpha\left(\frac{dP_3}{d\tau}\right) + \lambda, \tag{11}$$

where γ, α, and λ are parameters of $f(P_3)$ (Fig. 3); $dP_3/d\tau = 0$ in the state of steady flow, and when $f(P_3) = \lambda = \text{const}$, while for $dP_3/d\tau > 0$ there is a lower rate of entry of material into zone 3, so V_3 increases, whereas the converse applies for $dP_3/d\tau < 0$.

Therefore, the inertial, elastic, and dissipative components can give rise to periodic modes during isothermal infiltration, which involve changes in the bed structure and fluctuations in the gas flow rate. If in addition the material is wetted by a liquid, the fluidization region becomes wider [5], and the peak on the $\varphi(Q_3)$ curve becomes larger (AB'D' in Fig. 2). Production of heat in the bed tends to increase the resistance and favors these oscillatory states. The sequence of sections showing faster and slower motion constitutes a form of relaxation oscillation associated with structure change.

System (10) describes the periodic processes of relaxation type together with the higher-frequency acoustic oscillations; the analysis may be simplified in the following two limiting cases:

1. For $Ma_1 = 0$, $Ma_3 = 0$

$$\begin{gathered}(P_1 - P_3) = \left(k_1 + \frac{k_2}{R_3}\right) Q_1^2; \\ Ca_3 \frac{dP_3}{d\tau} = Q_1 - k_3 Q_3; \\ \frac{k_4}{3k_6} Q_1 = \frac{dR_3}{d\tau} R_3^2 + \frac{k_5}{3k_6} R_3^2 f(P_3), \\ P_3 = \varphi(Q_3).\end{gathered} \tag{12}$$

2. For $R_3 = \text{const}$

$$\begin{gathered}Ma_1 \frac{dQ_1}{d\tau} = (P_1 - P_3) - \left(k_1 + \frac{k_2}{R_3}\right) Q_1^2; \\ Ca_3 \frac{dP_3}{d\tau} = Q_1 - k_3 Q_3; \\ k_4 Q_1 = k_5 R_3^2 f(P_3); \\ P_3 = \varphi(Q_3) + Ma_3 \frac{dQ_3}{d\tau}.\end{gathered} \tag{13}$$

Linearization is facilitated by replacing the equations in (12) and (13) by equations in variations; then (12) and (13) take the same form for the steady state:

$$P_1 - P_{3,0} = \left(k_1 + \frac{k_2}{R_{3,0}}\right) Q_{1,0}^2; \quad Q_{1,0} = k_3 Q_{3,0};$$
$$k_4 Q_{1,0} = k_5 R_{3,0}^2 f(P_{3,0}); \quad P_{3,0} = \varphi(Q_{3,0}). \tag{14}$$

The following is the linearized form for the differential equation for the low-frequency relaxation oscillations:

$$\frac{k_2 Q_{1,0}^2}{R_{3,0}} R - \left(\frac{2k_2 Q_{1,0}}{R_{3,0}} + 2k_1 Q_{1,0}\right) Q - P = 0; \quad Ca_3 \frac{dP}{d\tau} = Q - k_3 q;$$
$$3k_6 R_{3,0}^2 \frac{dR}{d\tau} = k_4 Q + \alpha k_5 R_{3,0}^2 \frac{dP}{d\tau} - 2\lambda k_5 R_{3,0} R, \; P = \beta q. \tag{15}$$

Here $\beta = d\varphi(Q_{3,0})/dq$ is the coefficient to the second term in the expansion of $\varphi(Q_{3,0} + q)$ as a Taylor series:

$$\varphi(Q_{3,0} + q) = \varphi(Q_{3,0}) + q \frac{d\varphi(Q_{3,0})}{dq} = P_{3,0} + \beta q. \tag{16}$$

A series of operations in (15) gives us a second-order equation for the fluctuations in the gas flow rate:

$$\frac{d^2Q}{d\tau^2} + a_1 \frac{dQ}{d\tau} + a_2 Q = 0, \tag{17}$$

where

$$a_1 = \frac{3k_6 R_{3,0}^2 + \alpha k_5 k_2 Q_{1,0}^2}{6Ca_3 k_6 R_{3,0} Q_{1,0} (k_2 + R_{3,0} k_1)} + \frac{2\lambda k_5}{3k_6 R_{3,0}} + \frac{k_3}{\beta Ca_3} - \frac{k_2 k_4 Q_{1,0}}{6k_6 R_{3,0}^3 (k_2 + R_{3,0} k_1)}; \tag{18}$$

$$a_2 = \frac{k_3 R_{3,0}}{2\beta Ca_3^2 Q_{1,0}} \left(1 - \frac{1}{k_6}\right) + \frac{\alpha k_5 k_2 Q_{1,0}}{6\beta Ca_3^2 R_{3,0} (k_2 + R_{3,0} k_1)} (k_3 - 1) +$$
$$+ \frac{2\lambda k_3 k_5}{3Ca_3 R_{3,0} k_6 \beta} + \frac{\lambda k_5}{3Ca_3 k_6 Q_{1,0} (k_2 + R_{3,0} k_1)} - \frac{k_2 k_3 k_4 Q_{1,0}}{6Ca_3 k_6 R_{3,0}^3 \beta (k_2 + R_{3,0} k_1)}. \tag{19}$$

The linearized system for the high-frequency acoustic oscillations is

$$P = -Ma_1 \frac{dQ}{d\tau} - 2(k_2 + R_{3,0} k_1) \frac{Q_{1,0}}{R_{3,0}} Q;$$
$$Ca_3 \frac{dP}{d\tau} = Q - k_3 q; \quad P = Ma_3 \frac{dq}{d\tau} + \beta q \tag{20}$$

and this gives us the third-order equation

$$b_1 \frac{d^3Q}{d\tau^3} + b_2 \frac{d^2Q}{d\tau^2} + b_3 \frac{dQ}{d\tau} + b_4 Q = 0, \tag{21}$$

where

$$b_1 = \frac{1}{k_3} Ca_3 Ma_1 Ma_3; \tag{22}$$

$$b_2 = \frac{2Q_{1,0}}{k_3 R_{3,0}} Ca_3 Ma_3 (k_2 + R_{3,0} k_1) + \frac{Ca_3 Ma_1}{k_3} \beta; \tag{23}$$

$$b_3 = Ma_1 + \frac{2Q_{1,0} Ca_3}{R_{3,0} k_3} (k_2 + Q_{3,0} k_1) \beta; \tag{24}$$

$$b_4 = \frac{\beta}{k_3} + \frac{2Q_{1,0}}{R_{3,0}} (k_2 + R_{3,0} k_1). \tag{25}$$

The stability of the system of Fig. 1 with respect to relaxation and acoustic oscillations is governed by the magnitudes and signs of the coefficients in (17) and (21), which themselves are dependent on the coefficients K_i in (10), along with the acoustic masses Ma_1, Ma_3 and the acoustic capacitance Ca_3. The quantities $Q_{1,0}$ and $R_{3,0}$ determine the type of mode for which the stability conditions have to be solved. Here β is the slope of the $\varphi(Q_3)$ curve at the working point, and for $Q_3 < Q^B_3$ we have $\beta > 0$, while for $Q_3 > Q^B_3$ we have $\beta < 0$.

Linearization of (12) and (13) is equivalent to assuming that the oscillations are harmonic; this is clearly so for the acoustic oscillations, whereas a harmonic form is only a first approximation for the relaxation oscillations.

The solution to (17),

$$Q(\tau) = N \cos(\omega_1 \tau - \varphi) \exp(-\delta\tau) \tag{26}$$

serves to define the stability conditions for the hydrodynamic process in the presence of random low-frequency perturbations. The usual situation is

$$\frac{k_2 k_4 Q_{1,0}}{6k_6 R^3_{3,0}(k_2 + R_{3,0}k_1)} - \frac{2\lambda k_5}{3k_6 R_{3,0}} \simeq 0, \tag{27}$$

so low-frequency perturbations are damped if

$$\beta > -\frac{6k_3 k_6 R_{3,0} Q_{1,0}(k_2 + R_{3,0}k_1)}{3k_6 R^2_{3,0} + \alpha k_2 k_5 Q^2_{1,0}}. \tag{28}$$

Therefore, the AB branch of the $\varphi(Q_3)$ characteristic is the region of stable working states. Flow rates $Q_3 > Q^B_3$ transfer the working point to the descending branch BD of $\varphi(Q_3)$ and when

$$\beta < -\frac{6k_3 k_6 R_{3,0} Q_{1,0}(k_2 + R_{3,0}k_1)}{3k_6 R^2_{3,0} + \alpha k_2 k_5 Q^2_{1,0}} \tag{29}$$

the system shows persistent oscillations having the initial frequency $\omega_1 = 0.5\sqrt{4a_2 - a^2_1}$, which goes over asymptotically to the natural frequency $\omega = \sqrt{a_2}$. The initial conditions representing the induced perturbation, viz., $Q(0) = Q_{in}$ and $dQ(0)/d\tau = Q_{in}'$, determine the amplitude of the oscillations

$$N^2 = Q^2_{in} + \frac{(0.5a_1 Q_{in} + Q'_{in})^2}{a_2 - (0.5a_1)^2} \tag{30}$$

and the phase:

$$\varphi = \operatorname{arctg} \frac{0.5a_1 Q_{in} + Q'_{in}}{[a_2 - (0.5a_1)^2]\, Q_{in}}. \tag{31}$$

Therefore, the value of β given by (28) is to be taken as defining the stability limit to the flow of gas through the bed for given values of k_i, $R_{3,0}$, $Q_{1,0}$.

The stability of the system of Fig. 1 with respect to acoustic oscillations is defined by the Hurwitz criterion, viz., positive values for $b_i > 0$ and for the differences $(b_2 b_3 - b_1 b_4) > 0$. The relationship between the b_i and the sign-variable constant β is such that acoustic oscillations are damped for $\beta > 0$, whereas perturbations caused by relaxation effects maintain the acoustic oscillations for $\beta < 0$.

The solution to (21) is the sum of eigenvalues:

$$Q(\tau) = C_1 \exp(2\sigma\tau) + C_2 \exp[-(\sigma - i\omega)\tau] + C_3 \exp[-(\sigma + i\omega)\tau] = C_1 \exp(2\sigma\tau) + D\cos(\omega\tau + \alpha)\exp(\sigma\tau), \tag{32}$$

where σ and ω govern the aperiodic and harmonic components in the solution of (32), and these are related to solutions $y_1 = 2\sigma$, $y_{2,3} = -\sigma \pm i\omega$ to the incomplete characteristic equation

$$y^3 + \left(\frac{b_3}{b_1} - \frac{b_2^2}{3b_1^2}\right) y + 2\left(\frac{b_2}{3b_1}\right)^3 - \frac{b_2 b_3}{3b_1} + \frac{b_4}{b_1} = 0, \tag{33}$$

while the constants

$$C_1 = [(y_1 - y_2)(y_1 - y_3)]^{-1}; \quad C_2 = [(y_2 - y_1)(y_2 - y_3)]^{-1} = 0.5D\exp(i\alpha_{in});$$
$$C_3 = [(y_3 - y_1)(y_3 - y_2)]^{-1} = 0.5\, D\exp(-i\alpha_{in})$$

are determined by the initial conditions.

The analysis of the acoustic oscillations can be simplified substantially by considering an approximate hydrodynamic model; we put $Ma_3 = 0$, as this quantity is usually less than Ma_1 by two orders of magnitude, and then (21) gives

$$\frac{d^2Q}{d\tau^2} + \left[\frac{k_3}{Ca_3\beta} + \frac{2(k_2 + R_{3,0}k_1)}{R_{3,0}Ma_1} Q_{1,0}\right] \frac{dQ}{d\tau} + \left[\frac{2k_3(k_2 + R_{3,0}k_1)}{R_{3,0}Ma_1Ca_3\beta} Q_{1,0} + \frac{1}{Ma_1Ca_3}\right] Q = 0. \quad (34)$$

The solution to (34) gives the natural frequency as

$$\omega = \sqrt{\frac{1}{Ma_1Ca_3}} \sqrt{1 + \frac{2k_3(k_2 + R_{3,0}k_1)}{R_{3,0}\beta} Q_{1,0}}, \quad (35)$$

where the first cofactor in (35) is the frequency of the Helmholtz resonator for $\rho_1 \simeq \rho_3$:

$$\omega_0 = \sqrt{\frac{s_1\rho_3c_3^2}{\rho_1l_1V_3}} = c_3\sqrt{\frac{s_1}{l_1V_3}}, \quad (36)$$

and the second cofactor incorporates the interaction between the gas flow and the discrete structure around V_3.

Therefore, frequency analysis based on (26) and (32) allows one to relate the periodic acoustic oscillations and the relaxation oscillations in the granular bed to the variations in the gas flow rate.

NOTATION

P, pressure; Q, gas flow rate; V, R, volume and radius of cavity; l, s, length and cross section of element; $Ma = \rho l/s$, acoustic mass of gas flow; $Ca = V/\rho c^2$, acoustic capacitance of V; k_i, coefficients defined in (1)-(9); a_i, b_i, coefficients in (17), (21); γ, α, λ, parameters defined in (11); β, slope at the working point; ω, circular frequency; δ, damping factor; σ, aperiodicity parameter in (32); τ, time; ρ, $\varkappa$, c, density, adiabatic index, and speed of sound in gas. Subscripts (except for a_i and b_i) 1, 2, 3 correspond to the points in Fig. 1; subscript 0 represents the steady state; superscripts A, B, and D represent working points in Fig. 2.

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